ABSTRACT
Cohesive modelling provides a simple method to introduce a process region in models of fracture. It is computationally attractive since it blends into the structure of finite element programmes for stress analysis. The development of computational methods and applications of cohesive modelling has accelerated during recent years. Methods to measure cohesive laws have also been developed. One class of such methods is based on the path-independence of the $J$-integral.

By choosing a path encircling the cohesive zone, $J$ can be shown to be given by the area under the traction-separation relation for the cohesive zone. Using an alternative path, $J$ can in some cases be directly related to the applied load and deformation with relatively modest or no assumptions on the material behaviour. Thus, the cohesive law can be measured.

Methods to measure cohesive laws for different specimen geometries are presented. The methods are used to measure the cohesive law in peel, shear and mixed-mode for an adhesive layer. A new method to measure cohesive laws in shear is presented. The method is shown to give accurate data with a much smaller test specimen than earlier methods.

INTRODUCTION
Cohesive models of fracture originate with Barenblatt’s seminal paper from 1962 on the equilibrium of cracks, [1]. With a cohesive zone, fracture is modelled as a process where a surface in the material first forms a cohesive zone. The zone is later separated into two crack surfaces. In the cohesive zone, the prospective crack surfaces are held together by cohesive traction. In its most basic form, the cohesive traction is assumed only to be a function of the separation of the prospective crack surfaces. In 1987, it was demonstrated how cohesive elements can be developed and used within the structure of conventional finite element programmes [2,3]. Today, commercial finite element codes as ABAQUS and LS-Dyna incorporate cohesive elements and their use in analysing fracture is increasing. While the development of numerical methods has been intense, methods to experimentally measure cohesive laws has gained comparably smaller interest. One early and seemingly straightforward method to measure cohesive laws is presented in [4]. In that work, a butt-joint configuration is used together with a stiff testing machine to measure cohesive laws for wood adhesives. The idea is that the stress state is homogeneous in the adhesive layer and thus, the stress is simply given by the acting load divided by the cross sectional area. Unfortunately, the testing case is prone to instability. A different class of methods are suggested in [5,6].

In these methods, the classical Double Cantilever Beam (DCB) geometry is used which results in an inhomogeneous stress state, Fig. 1. By assuming the existence of a cohesive law, it is shown how the cohesive law can be measured.

\[ \sigma(w) = \begin{cases} \frac{2}{b} \frac{d}{dw} F \theta & \\ \frac{6}{b} \frac{d}{dw} \frac{d}{dw} M^2 & \end{cases} \] (1a,b)

where $w$, $E$, $b$, $h$ and $\theta$ are the elongation of the adhesive layer at the crack tip, Young’s modulus of the beam material, the out-of-plane width of the specimen, the height of the beams and the...
rotation of the cross section at the loading point, respectively. Note, that the same cohesive law \( \sigma(w) \) is assumed valid all along the complete joint. Another early method based on the \( J \)-integral is given in [6] based on Euler-Bernoulli beam theory. The Compact Tension Specimen (CTS) is used together with an approximate expression for \( J \) of the outer region. In [8] it is shown that the assumption of Euler-Bernoulli beam theory is unnecessary in deriving Eq. (1a) and in [9] the method is extended to large deformations. In [8,9], Eq. (1a) is derived using the \( J \)-integral. The papers mentioned above also demonstrate diverse applications of cohesive models: delamination of composites [5], fracture of adhesives [4,6], fracture of cementitious materials [7], and biological tissues [9]. Experimental results and different approaches to validate the methods are given in [4,7,8,10-13].

Measurement of cohesive laws in shear are treated in [5,14-17]. Some methods have also been developed to measure cohesive laws under mixed mode loading [18,19]. In shear, the end-notch flexure specimen has been used frequently though it sometimes leads to instability, [20-22] and Fig. 2.

**Figure 2:** End-notch flexure specimen.

Most of the methods mentioned above are dependent on a linear elastic behaviour of the beams. One exception is the DCB-method with transversal forces, cf. Eq. (1a) and Fig. 1. This method is independent of material properties, as long as they are homogeneous in the horizontal direction. This condition is a basic requirement for the path-independence of the \( J \)-integral. It is also required that a strain energy density can be identified for the loading case.

The requirement of elastic behaviour and a requirement to accommodate the non-linear part of the adhesive layer within the space between the crack tip and the loading point of the ENF-specimen, leads to large specimens [17]. With modern tough engineering epoxies a total length of up to 1 m is often required. It is obviously experimentally difficult to manufacture specimens with a well-controlled layer thickness with specimens of this length. Recently, toughness values so large that even larger specimens would be necessary have been developed. There is thus a need to develop experimental methods that allows for smaller and therefore plastically deforming beams.

In the present paper, we develop a new method based on the ENF-geometry that allows for plastic deformation of the beams. The plan of the paper is to first develop the theory for the \( J \)-integral based methods to measure cohesive laws. The design of the experiment is given in the third section and the experimental set-up in the following section. Some experimental results are given in the fifth section. The paper ends with discussions and conclusions.

**J-INTEGRAL METHOD**

The \( J \)-integral method is based on evaluating \( J \) along two alternative paths: one inner and one outer path. The \( J \)-integral is given by

\[
J = \int_{C_1} \left( U dy - T \cdot \frac{\partial u}{\partial x} dC \right)
\]

where \( U, T \) and \( u \) denote the strain energy density, the traction and displacement vectors, respectively. A Cartesian coordinate system with \( x \) along the adhesive layer is used. It is noted that the material behaviour must be independent of \( x \), i.e. \( U \) is not allowed to be explicitly dependent of \( x \). It is also noted that the material needs not be formally elastic as long as \( U \) can be defined for the specific loading state. Figure 3 shows the region at the start of the adhesive layer in a beam-like test specimen.

**Figure 3:** Crack region in beam-like test specimen and basic deformation modes of an adhesive layer with thickness \( t \).

Figure 3 also shows the basic deformation modes and conjugated stresses. These are denoted peel (\( \sigma, w \)) and shear (\( \tau, v \)). By evaluating Eq. (2) along the path \( C_1 \) according to Fig. 3, \( J \) is given by

\[
J(w, v) = \int_0^w \sigma(\bar{w}, \bar{v})d\bar{w} + \int_0^v \tau(\bar{w}, \bar{v})d\bar{v}
\]

In this way, \( J \) is shown to be given by the “consumed” strain-energy-release-rate at the crack tip. Another useful interpretation is that \( J \) is the strain energy per unit area captured in the adhesive layer at the crack tip.

The alternative integration path for the ENF-specimen is shown in Fig. 4.

**Figure 4:** End-notch flexure specimen with integration path \( C_2 \).
By using a properly designed overhang, cf. \( c \) in Fig. 4, \( J \) does not receive any contributions from the vertical paths at the right end of the specimen. Only the parts of the integration path \( C_2 \) where the load \( F \) and the reaction forces act contribute to \( J \). Evaluation of Eq. (2) shows that the contribution equals the product of the acting force and the rotation of the beams at the point the force acts. Equilibrium gives the reaction forces and by denoting the clockwise rotations of the three points where transversal forces acts \( \theta_1, \theta_2 \) and \( \theta_3 \) from left to right, respectively, \( J \) is derived as

\[
J = \frac{1}{b} [ (1 - \alpha) F \theta_1 - F \theta_2 + \alpha F \theta_3 ]
\]

(4)

If large deformations need to be considered, \( \sin(\theta) \) should replace \( \theta \) in Eq. (4) and the cohesive law should be considered as force per unit area in the un-deformed configuration, [9].

With the ENF-specimen the compressive peel stress at the crack tip is neglected and the cohesive law is shear driven by

\[
\tau = \frac{1}{b} \frac{d}{dv} [ (1 - \alpha) F \theta_1 - F \theta_2 + \alpha F \theta_3 ]
\]

(5)

Thus, if in an experiment, \( F, v, \theta_1, \theta_2 \) and \( \theta_3 \) are measured continuously, the cohesive law can be derived from Eq. (5). As noted from this expression, material data for the beams do not enter. Nor do we need to assume a specific material law when deriving it. This shows that the only assumption that needs to be considered is our ability to associate a strain energy density function to the behaviour of the beams. Thus, plastic deformation of the beams is allowed as long as influences of e.g. un-loading from a plastically deformed state are limited.

**DESIGN OF EXPERIMENT AND EXPERIMENTAL SETUP**

In the design of the experiment, we start by simulating the experiments. A finite element model (ABAQUS) consisting of beam elements (B21) for the adherends and cohesive elements (COH2D4) for the adhesive layer is used. Rigid elements (CONN2D2 of type BEAM) connect the nodes at beam centres with the nodes of the cohesive elements.

**Figure 5: Stress/deformation relation used in simulations.**

A cohesive model providing a bi-linear cohesive law in pure shear is used to model the shear behaviour of the engineering epoxy adhesive DOW-Betamate XW-1044-3 with a layer thickness of \( t = 0.2 \) mm, cf. Fig. 5. Cohesive data (\( \tau_{\text{max}} = 45.3 \) MPa, \( v_0 = 15.1 \) \( \mu \)m, \( v_c = 150 \) \( \mu \)m) correspond to values obtained by a different experimental method where the adherends are larger and deform elastically during the entire experiment, cf. [23]. These data give \( J_c = 3.4 \) kN/m which is the largest toughness measured for this adhesive. A mixed mode model is used in ABAQUS, but the peel stress remains small during the entire simulation and the precise nature of the mixed mode model is immaterial in this case.

The adherends are made of steel (Uddeholm, FORMAX) with an engineering stress strain curve as shown in Fig. 6. A von Mises isotropic hardening elastic-plastic model with tabulated values from the curve is used in the simulations.

**Figure 6: Stress strain curve for the adherend steel Uddeholm FORMAX.**

The aim is to design a specimen as small as possible with respect to the size needed for attachment of the measurement equipment. With this in mind the adherend thickness \( h = 6.6 \) mm, a span length of \( L = 200 \) mm and a width of \( b = 40 \) mm is decided, cf. Fig. 4. Furthermore, \( \alpha = \frac{1}{2} \) is used. An overhang of \( c = 50 \) mm is used in order to minimize the adhesive stress at the right end of the specimen and thereby ensuring the requirements for Eq. (4).

**Figure 7: Load displacement curve from simulation.**

Figure 7 shows the load displacement curve from FE-simulations. It is seen that yielding of the beams starts well before the maximum load is reached and that the force decreases somewhat before the crack starts to propagate. This is a source of error in the measurement of \( J \), since elastic unloading takes place from a plastic state. The solid curve in Fig. 8 shows the evolution of \( J \) as determined by Eq. (4); it is very close to the dashed curve showing the actual value of \( J \) as experienced by the adhesive at the crack tip, cf. Eq. (3). The
error induced by unloading from a plastic state is only 0.6 % at the start of crack propagation and 2 % when the crack has propagated 10 mm. Hence, the simulations indicate that the method to measure $J$ as given by Eq. (4) works well for the present configuration.

![Figure 8](image)

Figure 8: Evolution of $J$ during the simulated experiment. Solid curve: $J$ evaluated from loads and rotations, cf. Eq. (4); dashed curve: $J$ evaluated at crack tip, cf. Eq. (3).

In the experiments, a standard dual column electromechanical test machine is used with a three point bending fixture. As shown in Eqs. (4,5), the force, three rotations and the shear deformation at the crack tip have to be measured during an experiment. The force is measured with a standard force transducer, and the shear deformation with a linear variable differential transformer (LVDT), cf. [17] and Fig. 9.

An image system is used to measure the rotations. Three ribs are glued onto the specimen at the loading points, cf. Fig. 9. A monochromatic surface is arranged as a background and photos are taken of the ribs at regular time intervals during an experiment. A special purpose MATLAB program is developed to calculate the rotations of the ribs, [24]. This is done by identifying the horizontal positions of the ribs at two different vertical positions. With the horizontal positions denoted $n_{x_{\text{upper}}}$ and $n_{x_{\text{lower}}}$, respectively, and $n_{y}$ denoting the difference in horizontal position, the angle is given by

$$\beta = \arctan \frac{n_{x_{\text{upper}}} - n_{x_{\text{lower}}}}{n_{y}}$$

(6)

The rotation is given by the differences between the current and the original values of $\beta$. The load-point deflection $\Delta$ is also measured during the experiment using the transducer built-in to the testing machine.

EXPERIMENT

In the experiment, the loading point is given a prescribed displacement rate. In this first test of the method, we use a very slow loading rate in order to give ample time for data acquisition. The total load point displacement is maximized to 3.3 mm. Figure 10 shows load vs. load point deflection during the experiment. The maximum load 7.2 kN is achieved at $\Delta = 2.1$ mm. This load is about 8 % smaller than the maximum load in the simulation. Thus, the fracture energy of the adhesive is expected to be smaller than 3.4 kN/m used in the simulation.

![Figure 9](image)

Figure 9: Experimental set-up. Upper picture shows three ribs glued to the specimen; lower picture shows the opposite side with of the three-point bending configuration with the load cell at the top; loading through a cylinder and specimen resting on two cylinders. Lower picture also shows the LVDT used to measure $v$ at the crack tip.

Figure 11 shows experimental $J$ vs. $v$; $J$ is evaluated using Eq. (4). The curve starts with a characteristic parabolic shape corresponding to a linear elastic response of the adhesive. At about 140 $\mu$m, the curve reaches a plateau corresponding to the fracture energy $J_c = 2.1$ kN/m. This is on the low side as compared with previous experiments, cf. [25]. In [17], fracture energies between 2.1 and 3.2 kN/m are reported for the same adhesive layer. Figure 12 shows the derived cohesive law using Eq. (5). Since the experimental data contain some scatter, a $J$ vs. $v$ curve is first adapted using the least square method. The technique is described in more detail in e.g. [17].
DISCUSSION
A new method to measure cohesive laws in shear is developed. As compared with earlier methods, cf. e.g. [17], the present method allows for plastic deformation of the adherends and do not rely on accurate data for the properties of the adherends. The principal advantage of the present method is, however, the possibility to use much shorter specimens.

CONCLUSIONS
Cohesive models provide convenient methods to simulate fracture of adhesively bonded structures. The models are computationally attractive. A number of methods have been developed to deduce or measure the cohesive law during the past twenty years. Of these methods, the ones based on the path independent $J$-integral are especially attractive, principally those that do not rest on too specific assumptions on the behaviour of the material of the test specimen.

Here, a new method for measuring shear properties is developed. As compared with our earlier method, the present method allows for much smaller specimens and some plastic deformation of the adherends. Both simulations and experiments show promising results. The method can easily be extended to large deformations by replacing $\theta_i$ with $\sin(\theta_i)$ ($i = 1, 2, 3$), in Eqs. (4, 5).

NOMENCLATURE

\begin{itemize}
  \item $a$ \hspace{1em} Crack length \hspace{1em} [m]
  \item $b$ \hspace{1em} Out of plane thickness of adherends \hspace{1em} [m]
  \item $C$ \hspace{1em} Integration path \hspace{1em} [m]
  \item $F$ \hspace{1em} Load \hspace{1em} [N]
  \item $h$ \hspace{1em} Height of adherend \hspace{1em} [m]
  \item $J$ \hspace{1em} Energy release rate, $J$-integral \hspace{1em} [N/m]
  \item $J_c$ \hspace{1em} Fracture energy \hspace{1em} [N/m]
  \item $l, L$ \hspace{1em} Length of specimen \hspace{1em} [m]
  \item $M$ \hspace{1em} Bending moment \hspace{1em} [Nm]
  \item $n$ \hspace{1em} Geometrical position \hspace{1em} [m]
  \item $t$ \hspace{1em} Thickness of adhesive layer \hspace{1em} [m]
  \item $U$ \hspace{1em} Strain energy density \hspace{1em} [N/m$^2$]
  \item $v$ \hspace{1em} Shear deformation of adhesive layer \hspace{1em} [m]
  \item $w$ \hspace{1em} Elongation of adhesive layer in peel \hspace{1em} [m]
  \item $T$ \hspace{1em} Traction vector \hspace{1em} [N/m$^2$]
  \item $u$ \hspace{1em} Displacement vector \hspace{1em} [m]
  \item $\alpha$ \hspace{1em} Shape factor [-]
  \item $\beta$ \hspace{1em} Angle [-]
  \item $\Delta$ \hspace{1em} Load point displacement \hspace{1em} [m]
  \item $\sigma, \tau$ \hspace{1em} Peel and shear cohesive stresses \hspace{1em} [N/m$^2$]
  \item $\theta$ \hspace{1em} Rotation of loading point [-]
  \item $\theta_i$ \hspace{1em} Rotation of loading points ($i = 1, 2, 3$) [-]
\end{itemize}

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REFERENCES


